Universidade Federal de Sergipe
Departamento de Estatística e Ciências Atuariais
Disciplina: Inferência I
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Lista de Exercícios 2

2.1) Considere uma amostra aleatória $X_1, \dots, X_n$ de tamanho n. Obtenha a função de verossimilhança da amostra considerando as distribuições abaixo:

(a) Binomial(m, p): $P(X = x) = \binom{m}{x} p^x (1-p)^{m-x}, \quad x = 0, 1, 2, \dots, m$

(b) Geométrica(p) com $P(X = x) = pq^x$, $x = 0, 1, \dots$

(c) Geométrica(p) com $P(X = x) = pq^{x-1}$, $x = 1, 2, \dots$

(d) Binomial Negativa(r, p) com $P(X = x) = \binom{x+r-1}{r-1} p^r q^x$, $x = 0, 1, \dots$

(e) Binomial Negativa(r, p) com $P(X = x) = \binom{x-1}{r-1} p^r q^{x-r}$, $x = r, r+1, \dots$

(f) Uniforme(a, b): $f(x) = \frac{1}{b-a}, \quad a < x < b$

(g) Normal(μ, σ^2): $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty, -\infty < \mu < \infty, \sigma^2 > 0$

(h) Gama(α, β): $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0$

(i) Beta(a, b): $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \leq x \leq 1$

(j) Weibull(λ, a): $f(x) = \frac{a}{\lambda} \left(\frac{x}{\lambda}\right)^{a-1} e^{-(x/\lambda)^a}, \quad x \geq 0$

2.1) (a) $\left[\prod_{i=1}^{n} \binom{m}{x_i}\right] p^{\sum_{i=1}^{n} x_i} (1-p)^{nm-\sum_{i=1}^{n}}$

(b) $p^n q^{\sum_{i=1}^{n} x_i}$

(c) $p^n q^{\sum_{i=1}^{n} x_i - n}$

(d) $\left[\prod_{i=1}^{n} \binom{x_i+r-1}{r-1}\right] p^{nr} q^{\sum_{i=1}^{n} x_i}$

(e) $\left[\prod_{i=1}^{n} \binom{x_i-1}{r-1}\right] p^{nr} q^{\sum_{i=1}^{n} x_i - nr}$

(f) $\frac{1}{(b-a)^n}$

(g) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$

(h) $\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^n \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\beta\sum_{i=1}^{n} x_i}$

(i) $\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^n \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} \left(\prod_{i=1}^{n}(1-x_i)\right)^{\beta-1}$

(j) $\left(\frac{a}{\lambda}\right)^n \left(\frac{1}{\lambda^n}\prod_{i=1}^{n} x_i\right)^{a-1} e^{-\sum_{i=1}^{n}\left(\frac{x_i}{\lambda}\right)^a}$